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Sound structure of Ancient Greek Harmonia Modes:

Heterodyne resonance and differential bases

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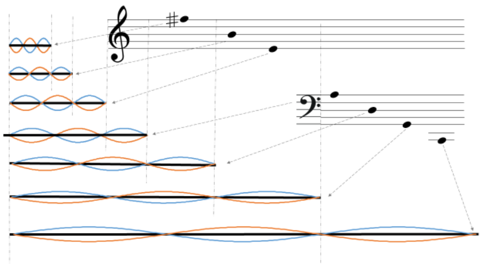
ABSTRACT

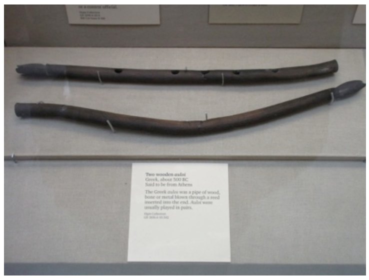
Role of Music was definitely significant among Police Societies of Ancient Greece. Ancient Greek Modes have such characteristic structures like “Tetrachord” or “Tonos”. Especially for “Harmonia Modes” are known as strange as containing micro tones smaller than half tones.

Having tried restoration of some ancient stringed instruments and reproduction of tunings according to Harmonia Modes, we are able to hear Difference Base Tone by twanging 2 strings which have difference with micro tone. This is a well-known differential resonance such as Heterodyne Modulation in Communication Engineering. We have solved the sound structure of ancient Greek Harmonia Modes, systematically, by reproducing them as combination of sine waves on computer.

1. Ancient Harmonia Modes with Architecture of Micro Tones

It is known that “Music” was one of the essential arts for liberal citizens in ancient Greek Police Societies [?]. Music Mode Theories in those days are generally consisted by two systems [?]: 1. Considering number ratio of Pythagoras school important, 2. Considering the "midpoint" (i.e. Difference according to Aristotle/Aristoxenos) important. Pythagoras Tuning centered on Perfect 5th. seems to be rational in contemporary view, but it is impossible to realize harmonic tones on A STRING, and problematic on tuning (Fig.1).





LEFT: **Figure.1.** Pythagoras Tuning is hard to be harmonic on A STRING.
RIGHT: **Figure.2.** "Aulos", B.C. 4-5 Century (British Musium).

We also show "Aulos" in Fig.2, as a wind instrument of Greece in B.C. 4-5 Century. Let's note that it is "midpoint" taken by empirically with Aristotle=Aristoxenos concepts, rather than "harmonic ratios", to drill the finger holes into the hollow tube of a tree. In ancient Greece, tunings are defined on length ratio of string (The idea "Frequency" had appeared after 19 C. with establishment of Fourier Analysis). The "length ratio" tuning is possible for only "octave", "4th", "5th", and "2nd" between 4th and 5th. For more detailed tones, those are tuned basically by "ears", and the discussion between Pythagoras-like numeric ratios and Aristoxenos-like differences was controversial for long years.
To show the geometrical relationships among "octave", "4th"and "5th", we reconstructed "HELIKON" [3] (Fig.3, 4)

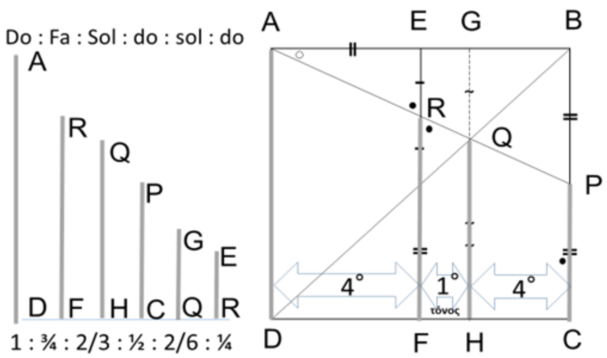


Figure.3. "Helikon" by Ptolemy, **Figure.4.** Reconstructed "Helikon"

Plotting like Fig.3, we can get both Perfect 4^{th}, one is for the ratio between line AD and RF (e.g. “do” and “fa”), the another for the ratio between line QH and PC (e.g. “sol” and “do (octave above)”). The difference between 4^{th} and 5^{th} constructed above is defined as “one tone”– Tonos, and this “one tone” is used to divide the 4^{th}. This structure for 4^{th} is called as “Tetrachord”. The Tonos has 8:9 ratio in frequency by simple calculation from contemporary view.

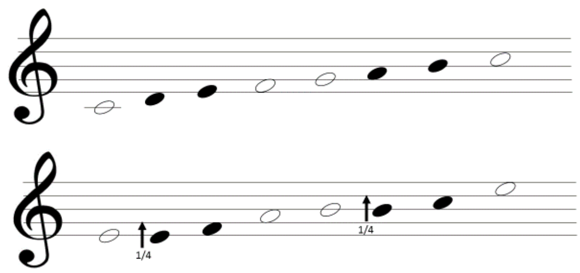


Figure.5. Tetrachords and an example of harmonia mode (Dorian)

The Upper side of Fig.5 shows a example of Aristoxenos-like Tetrachord. It places Tonos on “do” --- that is “re”, a Tonos placed on “re” --- that is “mi”, a “Half Tone” placed on “mi” --- that is “fa”. The Half Tone is considered as “a half of Tonos” qualitatively. It is obvious by simple calculation, this residual is NOT the “HALF” of “Tonos” both rationally and differentially. Therefore, Aristoxenos had been criticized later by Neo-Platonism school, etc. However it is clear by the example of “Aulos” pipe above, in musical practice, the framework of Aritsoxenos is very effective and it has influenced long time.

2. Harmonia Tuning And Quarter Tone

These “Tetrachord” structure, i.e. using 4 strings for Perfect 4^{th} are the “Harmonia Modes” widely spread in ancient Greek world, which divide the half tone divided more (It seems to be from an idea of “Moderation” of Aristotle).
For example, let’s try to make a Harmonia Mode built in Tetrachord structure like “Mi-La/Si-Mi”. The tone between “Mi” and “Fa” is a half tone in this context above, let’s insert a moderate string “Mi ↑ ” between Mi and Fa. It is also the half-tone relationship for “Si” and “Do”, let’s insert a moderate string “Si↑”. The resulting mode is shown as lower side of Fig.5.

These tones construct the several kinds of “Harmonia Modes” with specific sound behavior = “Mode”.
Through the vast literature research, Western Classical Philology interpreted that such temperament of modes with “quarter tones” is "Harmonia" = octave relationship. But the real sound effects of those are not shared well among the Western Classical Philologists.

In general, we can observe “beat” when we sound the proximate pitch strings at the same time. This is a physical sound-waves-interference, if we suppose the vibration of string as a sine wave, it is able to indicate the mechanism with a simple calculation. It is not only the case for proximate waves to observe the beat between them as physical entity vibration of difference between two frequencies. But also in several Engineering fields, mixing of two waves has been widely applied in such Heterodyne amplification. According to the ancient Greek scale above, it is 8:9 ratio in basic frequencies corresponding to two sounds as “Tonos”= whole tone, at the same time, then the base tone can sound corresponding to 1/8, i.e. 3 octaves below. In the same fassion, the differential fundamentals on diatonic scale are shown below. We can find most of constituting sounds for diatonic scale is “differential-harmonic”.

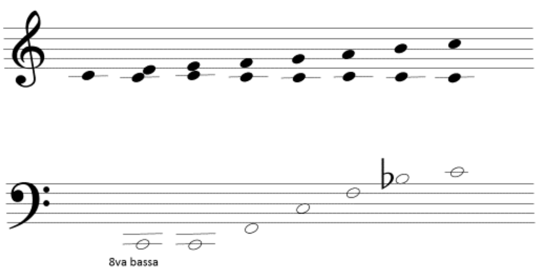


Figure.6. Differential fundamentals on diatonic scale

In the same fassion, by defining two tones which are half-tone or more narrow-pitch tone, it can sound the base tone like 1/16 to the original tone (4 octaves below), 1/32 (5 octaves below), or 1/64 (6 octaves below), at the same time.

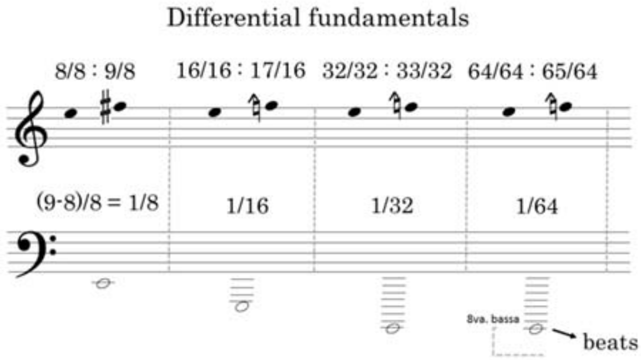


Figure.7. Differential fundamentals in Harmonia Mode [1]

What is the so-called "Discovery of Harmonia Modes" in ancient Greek world does not described well in detail in the literatures. But we can easily check the base tone by micro-tone tunings, with a simple calculation and reconstruction of the ancient instruments. The acoustic sounds of ancient Greek Harmonia Modes can be found as the use of the base tone compliant to Heterodyne phenomenon. For example, Dorian mode described by Aristides Quintilianus in A.D. 3 C., and Phrigian mode also described by him are shown below. Those 2 modes are from 6 modes described in Plato's "The Republic" in B.C. 4 C. [1].

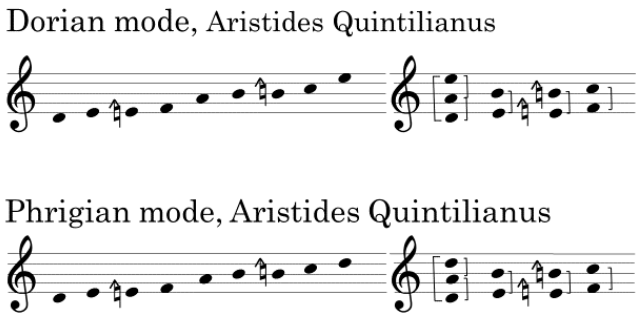


Figure.8. Examples of Harmonia Modes [1]

3. Mathematical Model with sine waves: Heterodyne effects

Let us examine the "difference tone" characteristics in the fundamental of Harmonia modes, by mathematical model based on sine waves. This is the approximation of physical sound phenomena.

Suppose there are two sine waves s_1 and s_2, corresponding to different frequencies f_1 and f_2 $(f_1 > f_2)$, where the amplitude is the same A, and each initial phases for both s_1 and s_2 are φ_1 and φ_2, *t* is time parameter.

$$s_1 = A \cdot sin(2\pi f_1 t + \varphi_1) \ \dots \ (1)$$

$$s_2 = A \cdot sin(2\pi f_2 t + \varphi_2) \ \dots \ (2)$$

Let us consider about the SUM of sine waves s_1+s_2. Suppose $\varphi_1 = \varphi_2 = 0$, $2\pi f_1 t = \alpha$, $2\pi f_2 t = \beta$ to ease the problem, we can find the well-known formula,

$$s_1 + s_2 = A \cdot \{\sin\alpha + \sin\beta\} = 2A \cdot \cos\frac{\alpha-\beta}{2} \cdot cos\frac{\alpha+\beta}{2} \ \dots \ (3)$$

Thus, we can get the other two sine waves generated by sinusoidal synthesis,

$$\frac{\alpha-\beta}{2} = 2\pi\frac{(f_1-f_2)}{2}t \ \dots (4), \qquad \frac{\alpha+\beta}{2} = 2\pi\frac{(f_1+f_2)}{2}t \dots (5)$$

for frequencies $\Delta_1 = \frac{f_1-f_2}{2}$ and $\Delta_2 = \frac{f_1+f_2}{2}$. Heterodyne Modulation is using this mechanism to modulate the signal wave. And this Δ_1 is exactly the base tone in Harmonia tuning. Actually, we can hear not the sound waves themselves but the work of them, we have to square the both sides, to consider those phenomena.

$$\{\sin\alpha + \sin\beta\}^2 = \sin^2\alpha + 2\sin\alpha\sin\beta + \sin^2\beta \ \dots (6)$$

4. Reconstruction of ancient instruments and difference base tone

Ancient Greek strings harp “Kithara” have been transmitted from B.C. 5-6 century to contemporary, and its resonance body is made of a shell of a turtle. It seems there was a mythical, magical or divine revelation sense. These “Kitharas” have over four strings, and tuned with several tetrachords, as estimated. Reconstructing the instrument like “Kithara” by contemporary resources, we can reproduce the difference base tone by Heterodyne effects.

However it is difficult to examine quantitatively for the real sound instrument with strings for vibration and resonance for a wood-box or a turtle shell. Thus, we made a choice to observe the differential base tone by the sound source with generated sine waves by computer and amplified the sound by Acrylic cavity. For these, we will report later.

Figure.9. Kithara (British Museum)

Figure.10. Reconstructed Kithara with contemporary material

Figure.11. Reconstructed Lyre Especially for base notes

5. Concluding Remarks

To examine these phenomenon above, we had selected the conditions below. For micro-difference tone, it is hard to tune precisely, we decided to generate it by PC digitally. And for its resonance, we decided to use a Cavity since it is easier to input the test sound source and record its output. It results a difference base tone and its harmonics, and the resulting graph is as below.

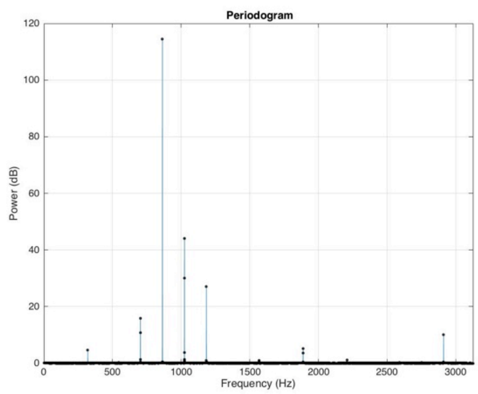


Figure.12.	**Figure.13.**

References:

[1] Barker, Andrew “The Science of Harmonics in Classical Greece”, Cambridge University Press., 2007.
[2] Barker, Andrew “Greek Musical Writings: I”, Cambridge University Press., 1984.
[3] Barker, Andrew “Greek Musical Writings: II”, Cambridge University Press., 1989.
[4] West, M.L., “Ancient Greek Music”, Oxford University Press., 1992.